On a New and Accurate Method of Determining Time, Latitude, and Azimuth with a Theodolite. By W. E. Cooke, M.A., Government Astronomer, Western Australia.

The *principle* of this method is not new. It was, I believe, first advocated by Mr. S. C. Chandler, who designed an instrument which he called an "Almucantar," developed the mathematical theory, and made a series of very fine observations at the Harvard College Observatory. An instrument of that kind, however, does not form part of a surveyor's ordinary outfit, whilst a theodolite does; and my present purpose is to show how the method may be applied to this universal instrument, and how by its means results of far greater accuracy than those usually obtained may be deduced. Briefly, the spirit level can be used instead of a mercury flotation in order to ensure constancy in zenith distance, and this brings the method within the power of every surveyor.

I shall first show it in connection with a 5-inch theodolite, and state the results of a few observations already made. Afterwards I propose to show that with a 12-inch instrument results can be obtained which, I believe, exceed in accuracy anything in field work hitherto published.

Five-inch Theodolite.

The observation for time and latitude consists in observing the time of transit of *Nautical Almanac* stars over the horizontal thread when the instrument is set so as to sweep a small circle in the sky parallel with the horizon, and at an altitude *about* equal to the observer's latitude. For azimuth it will be necessary, in addition, to take the time of transit across the vertical thread and to read the azimuth circle. An ordinary watch which possesses a seconds hand will do for timekeeper. The advantages may as well be enumerated here.

For time and latitude errors of vernier reading do not exist, as we require the circles for approximate setting purposes only. Extreme accuracy of construction and adjustment is unnecessary. We require only ordinary care in levelling. Error of collimation is absolutely immaterial. We do not even require that the bubbles shall be properly adjusted. All we require in the way of adjustment is that the vertical axis shall be reasonably vertical, so that the cross bubble on the altitude circle shall remain *fairly* steady as the instrument is swung round in azimuth, and great accuracy is not required even for this. As a matter of fact the instrument I used had been standing in a corner for months, and I simply took it out and levelled it up.

Yet, notwithstanding this apparent want of respect, results of considerable accuracy may be easily obtained. Those obtained in two evenings' trial will be stated almost immediately.

For azimuth I am afraid we must still depend upon the accuracy of the circles, and this immediately introduces errors which appear absurdly large compared with those for time and latitude. Incidentally it shows what an advance has been made by adopting this method. But even for azimuth it is confidently believed that the portion of error due to the star observation is greatly reduced, and that as the result of an evening's work the instrumental error is deduced with an accuracy far exceeding the possible setting. I may state here that I am not a practised observer with a theodolite, and possibly those who are may be able to read the circle itself with greater accuracy; but with a flickering candle-lamp I found it sometimes difficult to see any marked difference between two adjoining vernier divisions, and I think it is doubtful whether any surveyor would care to guarantee his accuracy within 1′ for each of a series of stars, some of the readings of which had to be made rather hurriedly. For azimuth, then, it can only be claimed that errors due to the star observation itself are probably reduced considerably, and that the observation can be conveniently made at the same time as the others, one setting being all that is required for each star, from which time, latitude, and azimuth are all deduced.

And now the results may be given for comparison with other methods, after which I shall show how the observations were made and reduced. I used a 5-inch theodolite, a candle lamp, and an ordinary watch. I had one vertical and one horizontal wire and no assistant. The watch did not tick either seconds, half-seconds, or any particular fraction of a second, and I had to get the time of transit the best way I could. I compared the watch during the evening with the standard sidereal clock, and thus I know its exact error. I observed on two evenings, with the following results:—

Time.

		Watch fast by direct comparison with sidereal clock.		Watch fast by observation of stars.	
		m	s	m	s
1902 Dec. 17	...	1	27·1	1	26·9
18	...	2	01·8	2	01·9

Latitude.

		°	′	″
Dec. 17	...	31	57	10·9
18	...	31	57	09·6

The real value of latitude is unknown to a fraction of a second, as the errors of division of the transit circle have not yet been determined; but it is, as nearly as I can obtain it, almost exactly midway between these two values. Such a remarkable agreement with a 5-inch instrument and common watch might be regarded as a mere coincidence were not its general accuracy determined by other considerations. It will, for instance, be found that the 12-inch gave quite unexpected agreement over

five nights. The principle upon which the observations are made produces confidence. And in particular the individual results show very satisfactory inter-agreement. I reproduce here those for the 18th, which are the better, as on the 17th I was new to the method and rather flurried at times.

Individual Results for each Pair, December 18.

Time.		Latitude.		
m	s	°	′	″
2	01·8	31	57	10·4
	1·5			9·0
	1·4			10·6
	2·3			8·4

The mean of the four watch errors does not agree exactly with that already given, but I had a ninth star unappropriated which was worked in.

I am, of course, willing to allow something for luck in such very close work; but I think it would not be possible to obtain anything like such results by the methods at present adopted.

Definitions.

I wish to define a few terms first. Let us adopt Chandler's name for the circle of reference. The almucantar, or small circle parallel to the horizon which passes through the celestial pole, will be called "the colatitude circle." There are in an ordinary 5-inch theodolite two screws which move the telescope in altitude when clamped. One moves it without disturbing any bubble. Let us call this one X. The other, usually consisting of a pair of antagonistic screws, moves both the telescope and the bubble to which the altitude circle is clamped. Let us call this screw Y, and it is particularly important to note the difference. X moves the telescope only, Y moves both telescope and bubble. This bubble we shall speak of as "the bubble." It is, in my instrument, the largest bubble of any except the stride, and extends from edge to edge of the vertical circle. It is the essential feature of the whole scheme, and the more sensitive it is the better. In mine one division equals 20″.

Other Definitions.

ϕ = observer's latitude, considered always positive.

α = star's R.A.

δ = star's declination, N being + and S −.

t = star's hour angle, + if west, − east.

θ = sidereal time of transit across colatitude circle.

A = star's azimuth at transit across colatitude circle.

Practice.

1. Prepare a working list of stars beforehand. Any stars down to the fifth magnitude may be taken from the *Nautical Almanac*. Two series are required. Let us call these

(*a*) *Prime vertical stars*, or those which cross the colatitude circle within about 20° N. or S. of the prime vertical. For Perth I take 0° to 30° south declination.

(*b*) *Latitude stars*, or those remote from the pole, whose hour angle is not greater than $2\frac{1}{2}$ hours, *i.e.* whose polar distance is within a few degrees, say 10°, of $(180°-2\phi)$. Of course it must be *less* than this quantity in order that the star may cross the colatitude circle.

Compute the sidereal time of observation and azimuth approximately by the following formulæ :—

$$\theta = a + t$$

$$\text{where } t = \begin{cases} \tan\phi \tan(45°-\frac{1}{2}\delta) \text{ for northern latitudes} \\ \tan\phi \tan(45°+\frac{1}{2}\delta) \text{ for southern latitudes} \end{cases}$$

and $$\tan\tfrac{1}{2}\mathrm{A} = \phi \tan t$$

using the best available value of ϕ for this purpose.

2. Set up and level instrument; in particular see that the level just mentioned is about the middle of its run. Set the telescope at an altitude $=\phi+$ refraction, using approximate values, of course the closer the better. Screw X may be used for this purpose, but after the first star is taken it must not be touched until the set is complete. A few minutes before the first star is expected, set for it in azimuth and clamp ; and now set "the bubble" quite accurately by means of screw Y. This must be done as carefully as possible. It is, in fact, the one essential adjustment, and a readjustment by screw Y must be made just before each star is taken. If when the star is seen in the field it appears that it will not cross the horizontal wire at the computed time the altitude may be altered by means of screw X ; but this applies to the first star only. The reason for the error will be an inaccurate telescope setting, or perhaps a large error in horizontal collimation. It may be adjusted approximately prior to the first transit, but afterwards must not be handled. It should perhaps be pointed out that this adjustment is purely a matter of convenience. The observations will give correct results even with a fairly large error of adjustment, but "the bubble" *must* be adjusted at some standard reading before each observation, the best being obviously the position of mid-level.

If the star appears to be moving in such a manner that it will not pass near the intersection of the wires the azimuth slow-motion screw may be used at any time.

3. Observe the times of transit across the horizontal and vertical wires, moving the azimuth screw so that the star crosses near, but not quite at, the centre. Do not touch the azimuth screw after transit of the vertical wire, or if you do, move it so that the star has to cross the wire again and take the second transit instead of the first. Then read the azimuth circle.

If only time and latitude are required take the horizontal transit only, and do not trouble about the azimuth circle.

Computation.

By means of the bubble we have observed the transit of a number of stars over the horizontal wire at some one definite zenith distance, *i.e.* over a definite almucantar, the accuracy of which depends upon the accuracy with which we have adjusted the bubble each time. We are supposed to have observed a few prime vertical and a few latitude stars, both east and west of the meridian, and if the number observed is equally divided between east and west so much the better.

From the prime vertical stars we shall find first the quantity Z, whereby the circle actually traced by the instrument differs from the true colatitude circle, and secondly the watch error.

From the latitude stars combined with Z, just obtained, we shall compute our latitude.

From any of the stars we shall compute the instrumental error in azimuth.

Our method requires that the latitude should be approximately known. If it is known with fair accuracy we may go straight ahead, but if not it will be as well to make a preliminary computation, taking one pair of prime vertical stars for Z and one pair for latitude. This will give us a result quite accurate enough for use.

Computation for Time.

With this value of latitude compute rigorously for each star values of θ and A by the formulæ already given, but use now seven-figure logs and take angles out to seconds at least. Compute also with four-figure logs

$Z = \operatorname{cosec} t \sec \delta$. Z is + for west and − for east stars in all latitudes. This quantity is required for every star.

$L = \dfrac{2}{15 \sin 2\phi} \cdot \cot t$. Same sign as Z. Required for latitude stars only.

If τ represents the observed time of transit reduced to sidereal time, form the quantity $(\theta+\tau)$ for each star. Distinguish eastern observations with an accent. Thus $(\theta-\tau)$ for west and $(\theta'-\tau')$ for east, &c. If the watch is fast this quantity will probably be negative, and if slow positive. Arrange the prime

vertical stars in two columns, west and east stars, and opposite each star place its Z and $(\theta-\tau)$. Take the mean of each of the four columns. We shall thus have mean values of Z, $(\theta-\tau)$, Z′, and $(\theta'-\tau')$, Z and Z′ being of opposite signs.

Then $$z = \frac{(\theta-\tau)-(\theta'-\tau')}{Z-Z'}$$

The watch error is practically the mean between $(\theta-\tau)$ and $(\theta'-\tau')$, but more acurately it equals

$$(\theta-\tau)-Zz = (\theta'-\tau')-Z'z$$

Be careful about the sign of z, as the correction to latitude depends upon this.

Each pair of stars may be treated separately as above, if it be required to see how they agree amongst themselves.

Computation for Latitude.

Enter the latitude stars in two columns, east and west, and opposite each star write its Z, $\theta-\tau$, and L.

Compute
$$a = (Z-Z')z$$
$$b = (\theta-\tau)-(\theta'-T')$$
$$c = L-L'$$

Then the correction to the latitude, or $\Delta\phi$, equals

$$\Delta\phi = \frac{b-a}{c}$$

and will be in seconds of arc. The sign obtained will be correct for either hemisphere, provided ϕ has been considered a positive quantity throughout; *i.e.* if $(b-a)$ is negative *subtract* $\Delta\phi$ numerically from the assumed value, and *vice versa*.

The computation may be applied to each pair separately, or to the mean of the east and mean of the west stars.

Computation for Azimuth.

Reduce the observed to sidereal time, apply the watch error, just obtained, and subtract this from the computed value of θ. Call this dt, and express it in seconds of time.

Compute $dA = 15 \sin A \cot t \,.\, dt$, using 4-figure logs, and apply this, which will be in seconds of arc, as a correction to the observed circle reading. The result will be instrumental azimuth, and the difference between it and A will be $= a$, the instrumental error in azimuth.

Observation with a 12-inch Theodolite.

The principle is the same as with a 5-inch. The modifications are :

1. The bubble is much larger. It is attached loosely, so as to revolve round the horizontal axis of the theodolite. It is set approximately level after the telescope-pointing in altitude has been made, and is then clamped firmly to the telescope. It is not adjusted before each observation. In fact, no alteration in altitude is made during the evening, but the bubble is read immediately after each transit, and a correction is made to reduce all observations to some definite level reading, arbitrarily chosen. One division is equal to $1''\cdot 2$.

2. Instead of one horizontal and one vertical wire there are, in my instrument, three horizontal and five vertical ones. I make a point of obtaining transit over each of the three horizontal and as many as possible of the vertical wires.

3. The mean of the vertical wires is not necessarily in the line of collimation, but for azimuth a micrometer is used and the reading for collimation obtained. I have tried several methods, but none of them are perfectly satisfactory, in my opinion. This, however, is a trouble common to any method for obtaining azimuth, and one of the great charms of this new method of determining time and latitude is that collimation lines and circle readings are things of the past. For azimuth, however, all transits must be reduced to the vertical line of collimation.

4. A chronometer, or chronograph if available, is now used, and the results are sufficiently accurate to require a correction for diurnal aberration.

Computation.

Instead of repeating the whole method of computation I shall confine myself to indicating the change caused by the variation in the instrument.

1. Correction for bubble-reading. The value of one division of the scale must be known. Some definite bubble-reading is to be arbitrarily selected, preferably nearly the same as the mean of all the readings taken, and the difference between this standard and the observed reading, expressed in arc seconds, must be multiplied by

$$\frac{dt}{d\zeta} = \frac{1}{15 \cos \phi \sin A}$$

The result, which will be in seconds of time, must then be applied as a correction to the observed mean time of transit over the horizontal wires.

No rules for signs can be given, as these will depend upon the method of graduating, &c., and may be different in different instruments. The sign for west stars will be opposite that for east.

2. Reduction to mean of wires or line of collimation. For this purpose the interval between the mean of observed times and (*a*) the mean of wires, for horizontal, (*b*) the line of collimation, for vertical wires, must be known. To save trouble I made a point of always obtaining the three horizontal transits, and simply took the mean of the three. But this may not always be practicable, so the following formulæ are given :

Let f = angular interval of mean of observed from the ideal line to which all stars are to be referred.

F = correction to be applied.

Then for horizontal transits

$$F = Zf + \tfrac{15}{2} f^2 \sin 1'' \, . \, Z^2 \cos t$$

and for the prime vertical set of stars only the first term need be used.

For transits over vertical threads

$$F = f \sec \phi$$

3. The correction for diurnal aberration is

$$\kappa = 0^{s}{\cdot}0207 \sin \phi$$

This is to be *added* to the computed clock-error, or *subtracted* from the observed time of transit.

4. A correction for rate of chronometer should be applied to each transit, to reduce to some definite sidereal hour.

Results.

Stars were observed on five evenings, time being taken by the eye and ear method with a sidereal chronometer beating half-seconds. The chronometer was compared before and after the evening's work with the standard sidereal clock. The theodolite was placed upon a small brick pillar in the meridian of the transit circle, and the figures below show the differences between the errors of the chronometer deduced from comparison with a sidereal clock and those by direct observation with the theodolite. The error of the sidereal clock was obtained by means of transits observed with the transit circle.

	Time. Diff. *TC*—theodolite.	Latitude.
	s	° ′ ″
1902 Dec. 8	−0·03	31 57 10·3
9	+0·01	10·1
10	−0·07	10·5
12	+0·02	10·2
13	0·00	10·2

A sufficient number of observations have not been made to test azimuth results, but such as they are they appear satisfactory. The only rough and ready method that occurred to me was to take the instrumental error in azimuth for each night as deduced from prime vertical and latitude stars respectively. Only two sets are available, as it did not occur to me to try this method for azimuth until the 12th, and even then I adopted different tactics for the two sets. For the prime vertical stars I kept the star bisected on the middle vertical wire with the slow-motion screw in azimuth until it reached a fixed horizontal wire, and took no time transits at all, reducing by a different method. I did not like this way, however, as well as the time transit, and only used it on that one occasion; but it is satisfactory to note the agreement between the two.

		P. Vert. Stars.	Latitude Stars.
Dec. 12	Inst. error in az.	$+10''.2$	$+10''.7$
13	,, ,, ,,	$+4.8$	$+4.8$

Of course the change between the 12th and 13th is of no account. That was almost certainly a real change.

In conclusion I must express my great obligation to S. C. Chandler for his work on the Almucantar published as vol. xvii. of the *Annals of Harvard College Observatory*, to which I refer all those who wish to see more of the mathematics and results of this beautiful method.

On the Sun's Stellar Magnitude and the Parallax of Binary Stars. By J. E. Gore, M.R.I.A.

The Sun's stellar magnitude, or the number which represents the Sun's brightness as seen from the Earth on the same scale as the "magnitudes" of the stars are represented, has been variously estimated at numbers ranging from -25.5 to -27.0. The following method of computing its value from the apparent brightness of binary stars, whose orbits are well determined and whose spectra resembles that of the Sun, has not, so far as I am aware, been previously published.

Let P = period of binary star in years

a = semi-axis major of orbit in seconds of arc

p = parallax of binary

Then we have

$$\text{Mass of binary system} = \frac{a^3}{p^3 P^2} \quad (\text{Sun's mass} = 1)$$